# Short Papers

## Microstrip Line Filters Using Yttrium Iron Garnet Film

Makoto Tsutsumi and Satoshi Tamura

***Abstract*—The yttrium iron garnet (YIG) film microstrip line is fabricated by using 40 μm thick film of width of 20 mm and length of 10 mm with a microstrip of width 0.7 mm, and magnetized in a transverse direction to the wave propagation. Sharp notch characteristics of more than 30 db with few dB insertion loss are observed experimentally with a variable center frequency from 9 GHz to 11 GHz. Results were explained phenomenologically with the coupled mode theory.**

### I. Introduction

There is now considerable interest in the magnetostatic wave in yttrium iron garnet (YIG) films grown epitaxially on gadolinium gallium garnet (GGG) substrate, since they promise of new class of microwave ferrite devices, which could be used for signal processing, such as resonators, filters, oscillators and delay lines [1], [2]. These devices are analogous to these which form the basis of surface acoustic wave technology.

In this short paper the band rejection filter is proposed as an application of the cut off behavior of microstrip line using YIG film substrate. Sharp notch characteristics are demonstrated experimentally at $X$ band. The filter characteristics were discussed with coupled mode theory between TEM and magnetostatic surface wave (MSSW) modes.

### II. Experimental Results

The experimental set up is shown in Fig. 1. It consists of 40 $\mu$m thick YIG film grown epitaxially on the GGG substrate of thickness of 400 $\mu$m. The dimension of YIG film is 10 × 20 mm$^2$, the strip width is 0.7 mm and the length of strip is 10 mm. Experiment was carried out at $X$ band. An interest is to incline the dc magnetic field in the $X$-$Z$ plane as shown in Fig. 1. The transmission characteristics of the strip line are shown in Fig. 2 for various values of the inclination angle $\theta$ of the magnetic field. A weak cut-off behavior of 12 dB is observed at $\theta = 0°$. When $\theta$ is increased up to 40°, sharp cut-off characteristic of more than 30 dB is observed. By changing the dc magnetic field the center frequency of the notch characteristic at $\theta = 40°$ is varied from 9 GHz to 11 GHz with few dB insertion loss. These represent a good frequency response of a variable band rejection filter at microwave frequency. The nonreciprocal filter characteristic is also observed by changing the magnetic field direction.

The filter characteristic depends on the strip width. The wide strip line of more than 1.2 mm has a tendency to increase ripples and spreads out the 3 dB bandwidth of filter. This can be estimated as the excitation of higher order mode in the width direction of the microstrip.

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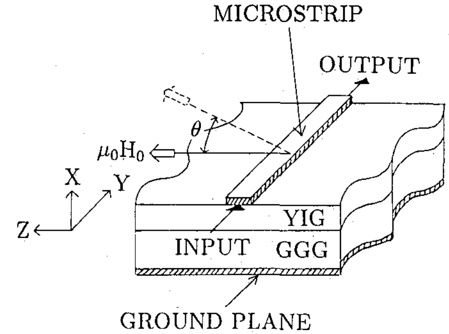


Fig. 1. Geometry of the YIG film microstrip line magnetized arbitrarily in the $x$-$z$ plane.

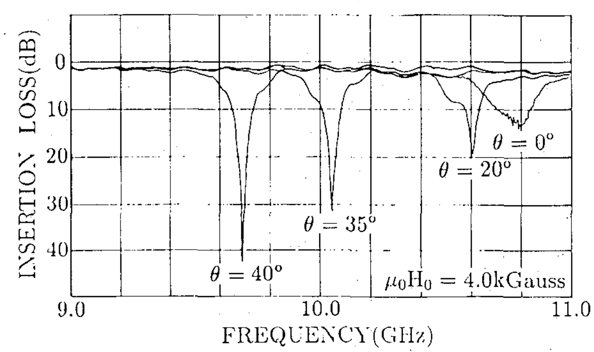


Fig. 2. Typical filter characteristic of notch type for various values of inclination angle $\theta$ of dc magnetic field directions.

### III. Discussions

Experimental results are discussed with coupled mode theory [3], [4]. The coupled mode theory is an attempt to explain experimental results, and is based on a parallel plate transmission mode of a inset of Fig. 3 because of the thin film structure of substrate. Thus microstrip line of Fig. 1 is approximated by a layered structure consisting a YIG film and a dielectric GGG substrate. In magnetostatic wave transducer problems the wave radiates right angle to the microstrip line, i.e., $Z$ direction, and the quasi-static theory was available [5], [6]. In the parallel plate transmission line mode magnetostatic approximation is not available, because the electromagnetic quasi-TEM mode is the dominant mode of the microstrip line and propagates in the $Y$ direction.

If a dc magnetic field is applied in the $Z$ direction of Fig. 3, TM and TE modes exist separately in such a two dimensional geometry [7], [8]. While if the dc magnetic field is inclined in angle $\theta$ in the $X$-$Z$ plane as shown in Figs. 1 and 2. TM mode couples to TE mode through the additional non-diagonal components of the permeability tensor of the YIG.

Permeability tensor of the ferrite magnetized in the $X$-$Z$ plane is given by

$$\hat{\mu} = \begin{bmatrix} \mu_{11} & j\mu_{12} & \delta\mu_{13} \\ -j\mu_{21} & \mu_{22} & j\delta\mu_{23} \\ \delta\mu_{31} & -j\delta\mu_{32} & \mu_{33} \end{bmatrix}, \tag{1}$$

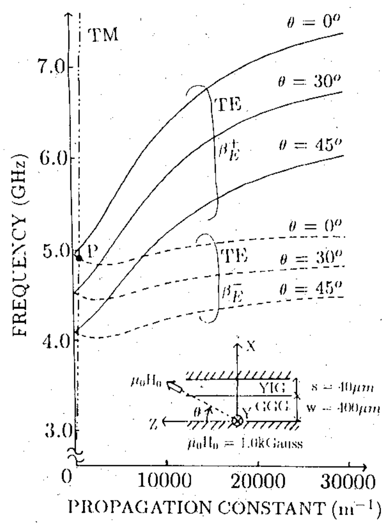


Fig. 3. Dispersion curves for various values of inclination angle $\theta$ of dc magnetic field direction.

where

$$\mu_{11} = \mu \cos^2 \theta + \sin^2 \theta,$$

$$\mu_{22} = \mu,$$

$$\mu_{12} = \mu_{21} = \kappa \cos \theta,$$

$$\mu_{23} = \mu_{32} = \kappa \sin \theta,$$

$$\mu_{13} = \mu_{31} = (1 - \mu) \sin \theta \cos \theta,$$

$$\mu_{33} = \cos^2 \theta + \mu \sin^2 \theta,$$

and

$$\mu = \frac{\omega_0^2 - \omega^2}{\omega_h^2 - \omega^2}, \quad \kappa = \frac{\omega \gamma \mu_0 M_0}{\omega_h^2 - \omega^2}, \quad \omega_h = \gamma \mu_0 H_0,$$

$$\omega_0 = \gamma \mu_0 \sqrt{H_0(H_0 + M_0)},$$

$\delta$ is smallness factor on $\theta$ and is unity for numerical calculation.

When $\delta$ is zero, Maxwell's equation leads to separate TM and TE modes. From boundary conditions of continuity of tangential electric and magnetic field at interface between YIG film and GGG substrate and zero component of tangential electric field on the parallel plates, the dispersion relations of both TM and TE modes are obtained and are evaluated numerically as shown in Fig. 3 for various values of the inclination angle $\theta$ of the dc magnetic field. In figure $\beta_E^+$ and $\beta_E^-$ of the TE mode denote the dispersion curves of propagation mode of $+y$ and $-y$ directions, respectively, which is similar curve of the MSSW mode influenced by the metal plate [8]. It is found that $\beta_E^-$ shows a negative group velocity below $\beta = 10000\ \text{m}^{-1}$. For large $\theta$ value more than $\theta = 30°$ volume wave mode appears below 4.6 GHz.

The dispersion curve of the TM mode is also shown in Fig. 3 which is almost same curve of TEM mode. The dispersion curve of the TEM mode intersects these of the TE (MSSW) mode at point $P$ showing phase matching. Thus, if the magnetic field direction $\theta$ is increased, maximum coupling between the two modes occurs at the phase matching point $P$ through the additional non-diagnonal components of the permeability tensor $\delta\mu_{13}$ and $\delta\mu_{23}$ of (1).

Coupled mode equation from TM (TEM) to TE (MSSW) modes can be derived with the help of orthogonal relation of modes as [3], [4]

$$\frac{\partial A_m(y)}{\partial y} = -jC_{ab} B_m(y) e^{\mp j\Delta y}, \tag{2a}$$

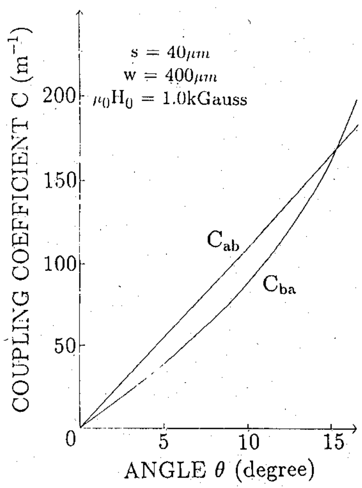


Fig. 4. Coupling coefficients as a function of the inclination angle $\theta$ of dc magnetic field direction

where the coupling coefficient is

$$C_{ab} = \frac{\delta AB}{4(\mu_{11}\mu_{22} - \mu_{12}^2)} [C_o \pm (\mu_{13}\mu_{12} + \mu_{23}\mu_{11}) \sinh k_x' s],$$

$$C_o = \frac{\beta}{k_x'} (\mu_{22}\mu_{13} + \mu_{12}\mu_{23})(\cosh k_x' s - 1),$$

$$k_x' = \sqrt{\frac{1}{\mu_{11}} [\beta^2 \mu_{22} - \omega^2 \epsilon_0 \epsilon_r \mu_0 (\mu_{11}\mu_{22} - \mu_{12}^2)]},$$

$s$ is the thickness of YIG film, $\Delta$ is the phase difference between TEM and TE modes and $\beta$ is the propagation constant.

Coupled mode equation from TE to TEM mode yields

$$\frac{\partial B_m(y)}{\partial y} = jC_{ba} A_m(y) e^{\pm j\Delta y}, \tag{2b}$$

where

$$C_{ba} = \frac{\delta AB}{4(\mu_{11}\mu_{22} - \mu_{12}^2)} \left[C_o \pm \frac{\mu_{12}}{\mu_{22}} (\mu_{13}\mu_{22} + \mu_{12}\mu_{23}) \sinh k_x' s\right].$$

Arbitrary constants $A$ and $B$ in the coupling coefficients $C_{ab}$ and $C_{ba}$ are defined by the normalization of the guided wave power.

Coupling coefficients $C_{ab}$ and $C_{ba}$ of (2a) and (2b) are evaluated numerically as a function of the inclination angle $\theta$ of dc magnetic field at $P$ and are shown in Fig. 4. Numerical values chosen are; thickness of YIG film $s = 40\ \mu\text{m}$, thickness of GGG, 400 $\mu$m, same dielectric constant $\epsilon_r$ of YIG and GGG, $\epsilon_r = 15.3$, $\mu_0 M_0 = 0.173\ \text{Wb/m}^2$, $\mu_0 H_0 = 0.1\ \text{Wb/m}^2$.

Small difference between $C_{ab}$ and $C_{ba}$ values is found in the figure. This is due to the approximation technique based on the coupled mode theory.

In the experiment, a weak cut-off behavior is observed at $\theta = 0°$, which implies that there is no coupling between two modes in the parallel plate transmission line approximation. It is predicted that the cut-off behavior might be caused by week coupling between TE and TM modes due to finite width of the strip [9].

In Fig. 4, at $\theta = 15°$ coupling coefficient of 200 $\text{m}^{-1}$ is observed which is sufficient to satisfy the perfect coupling condition; $c|L| = \pi/2$, where $L$ is the length of YIG film, which is 10 mm for the sample used in the experiment. Hence, the cut-off behavior observed at $\theta = 20°$ is caused by the perfect coupling between TEM and MSSW modes which occur at the phase matching point $P$.

A good cut-off behavior at $\theta = 40°$ is observed in Fig. 2. The coupled mode theory at such a large $\theta$ value fails due to the approximation. However, strong coupling phenomenon is extrapolated from Fig. 4, which spreads out the stop bandwidth around point $P$. The resonant frequency of MSSW ($\omega \simeq \omega_h + \gamma\mu_0 M/2$) may be also included within stopband. Thus coupled power is absorbed by the resonance behavior and sharp cut-off of more than 30 dB may be observed.

## IV. Conclusion

We have proposed a band rejection filter which uses the coupling between TEM and MSSW modes in the YIG film microstrip line. Experiments have been performed by using 40 $\mu$m thick YIG film, 400 $\mu$m thick GGG and 0.7 mm width strip, and for various dc magnetic field directions. Sharp notch characteristics of more than 30 dB have been observed at $X$ band, and results are phenomenologically explained with a coupled mode theory.

These characteristics are very useful for magnetostatic wave application to band rejection filters at microwave frequency.

## Acknowledgment

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# Variational Method for the Analysis of Lossless Bi-Isotropic (Nonreciprocal chiral) Waveguides

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*Abstract*—**Equations are derived for the longitudinal fields of a propagating mode in the most general straight open waveguide structure made from the most general lossless linear material whose parameters are independent of its orientation. This material, also called bi-isotropic, has the important chiral medium as the reciprocal special case. Self-adjointness of the differential operator with respect to the hermitian inner product is confirmed. Applying the theory of nonstandard eigenvalue problems, a variational expression is derived for the solution of the waveguide problem. A procedure for its application is discussed.**

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## I. Introduction

The chiral medium has raised considerable theoretical interest in recent years because of its unique properties. In fact, offering an extra parameter, it gives the possibility of satisfying conditions beyond those of isotropic media. Among suggested applications we may mention realization of reflectionless surfaces, for which there exist numerous patents [1], creating polarization rotating microwave devices without ferrites [2], and interesting antennas, like closely packed microstrip antennas with less coupling between the elements [3]. Also there exists a monograph on electromagnetics in chiral media [4].

The "chiral medium" treated in the literature has mostly been reciprocal. However, accepting nonreciprocity, we have one more parameter to deal with and the medium can be called nonreciprocal chiral or, more generally, bi-isotropic. Such a medium without chirality was introduced by Tellegen in 1948 to realize a new circuit element called the gyrator [5]. Wave propagation in a bi-isotropic medium was recently studied by J. C. Monzon [6]. The medium equations can be written in the form [7]

$$\boldsymbol{D} = \epsilon \boldsymbol{E} + \xi \boldsymbol{H}, \tag{1}$$

$$\boldsymbol{B} = \mu \boldsymbol{H} + \zeta \boldsymbol{E}, \tag{2}$$

with

$$\xi = (\chi - j\kappa)\sqrt{\mu_o \epsilon_o}, \quad \zeta = (\chi + j\kappa)\sqrt{\mu_o \epsilon_o}. \tag{3}$$

For lossless media, the parameters $\kappa$ and $\chi$ together with $\epsilon$ and $\mu$ are real, whence $\xi = \zeta^*$, which case is assumed here. The chirality parameter $\kappa$ gives the rate of polarization rotation of a linearly polarized plane wave, relative to the rate of phase change of the wave in propagation. The Tellegen parameter $\chi$ is proportional to the polarization rotation of a plane wave in reflection from a discontinuity [7].

In the present paper, the electromagnetic problem of wave propagation along a general straight open waveguide of bi-isotropic material is formulated in terms of a variational method. A similar method has been previously derived and applied for dielectric and corrugated waveguides and labeled as 'a variational method for nonstandard eigenvalue problems' [8]–[11]. The nonstandard, or nonlinear, eigenvalue problem is one which can be written in the general operator from $L(\lambda)f = 0$, where the eigenvalue parameter $\lambda$ is not necessarily a linear coefficient. The result of this way of thinking [8] is that any parameter of the problem can be taken as the eigenvalue parameter $\lambda$ and if it can be solved analytically from a functional equation, what results is a stationary functional for that parameter. This is also the procedure followed in this paper for the bi-isotropic problem.

## II. Theory

The bi-isotropic medium considered here is homogeneous along one space direction labeled with the $z$ coordinate and a function of the transverse position vector $\boldsymbol{\rho} = \boldsymbol{r} - \boldsymbol{u}_z(\boldsymbol{u}_z \cdot \boldsymbol{r})$. Let us asume that the structure is concentrated close to the $z$ axis and the param-